LANDAU PARAMETERS for FINITE RANGE INTERACTION

following D. DAVESNE, A. PASTORE, J. NAVARRO arXiv 1401.7914v1 Appendix A

$$\sum_\ell f_\ell^{(\alpha)} P_\ell(\cos\theta) = V_{ph}^{(\alpha)} = D^{(\alpha)} - E^{(\alpha)}$$

for $(\alpha) = (0,0)$ isoscalar-scalar channel

$$\sum_\ell f_\ell P_\ell(\cos\theta) = D^{(0,0)} - E^{(0,0)} = \tilde{W} + \frac{1}{2}\tilde{B} - \frac{1}{2}\tilde{H} - \frac{1}{4}\tilde{M} - \left(\frac{1}{4}\tilde{W} + \frac{1}{2}\tilde{B} - \frac{1}{2}\tilde{H} - \tilde{M}\right) = \underline{\frac{3}{4}\left(\tilde{W} + \tilde{M}\right)}$$

tilde on Fourier transformed terms

for Skyrme zero Range $\tilde{M} = \tilde{H} = 0$ since the isospin terms are trivial

$$V_{Skyrme} = t_0(1 + x_0 P^\sigma) + t_1(1 + x_1 P^\sigma)\frac{1}{2}(k^2 + k'^2) + t_2(1 + x_2 P^\sigma)\,\vec{k}\cdot\vec{k}'$$

$$V_{sk\text{-}ph} = V_{Skyrme}\,(1 - \hat{P}^x \hat{P}^\sigma \hat{P}^\tau)_{1,2} \qquad \text{because } \frac{\delta^2 E}{\delta\rho_1 \delta\rho_2} = V(1 - P^x P^\sigma P^\tau)$$

with 0 range Skyrme $\hat{P}^x \hat{T}_1 = \hat{T}_1$, $\hat{P}^x T_2 = -T_2$; $P^x P^\sigma P^\tau = -1$ on antisymmetrized states

for Skyrme $\hat{P}^x = \pm 1$ (depending on the term)

EXPLICIT

$$V_{sk\text{-}ph}^{(0)} = t_0(1 + x_0 P^\sigma)(1 - P^x P^\sigma P^\tau) =$$
$$= t_0(1 + x_0 P^\sigma)(1 - P^\sigma P^\tau) = t_0(1 - P^\sigma P^\tau) + x_0(P^\sigma - P^\tau)$$
$$= t_0\left(1 - \frac{1}{4}(1 + \vec{\sigma}\cdot\vec{\sigma})(1 + \vec{\tau}\cdot\vec{\tau})\right) + x_0 t_0\left(\cancel{1} + \vec{\sigma}\cdot\vec{\sigma} - \cancel{1} - \vec{\tau}\cdot\vec{\tau}\right)\frac{1}{2}$$
$$= \frac{3}{4}t_0 - \frac{1}{4}(t_0 - 2x_0 t_0)\,\vec{\sigma}\cdot\vec{\sigma} - \frac{1}{4}(t_0 + 2x_0 t_0)\,\vec{\tau}\cdot\vec{\tau} - \frac{1}{4}t_0(\vec{\sigma}\cdot\vec{\sigma})(\vec{\tau}\cdot\vec{\tau})$$

$P^\sigma = \frac{1}{2}(\delta + \vec{\sigma}\cdot\vec{\sigma})$

$P^\tau = \frac{1}{2}(\delta + \vec{\tau}\cdot\vec{\tau})$

from PRC 65, 014316 Pudliner (2006) Eq. (53)

USING RELATION

$$f^{(0,0)} = D^{(0,0)} - E^{(0,0)} = \frac{3}{4}\tilde{W} = \frac{3}{4}t_0 \qquad f^{(1,0)} = \frac{1}{2}\tilde{B} - \frac{1}{4}\tilde{W} = -\frac{1}{4}t_0(1 - 2x_0)$$

$$f^{(0,1)} = -\frac{1}{4}\tilde{W} - \frac{1}{2}\tilde{B} = -\frac{1}{4}t_0(1 + 2x_0) \qquad f^{(1,1)} = -\frac{1}{4}\tilde{W} = -\frac{t_0}{4}$$

Higher Order

Legendre Polynomial $P_\ell(x) \to P_0(x) = 1$
$P_1(x) = x$
$P_2(x) = \frac{1}{2}(3x^2 - 1)$ ...

$\Rightarrow$ Ist order

t_1 is analogous to t_0

I^{st} order Skyrme Landau Parameters

t_1 is analogous to t_0

t_2 terms imply $P^x \; k_{12}^*\cdot k_{34} = -k_{12}^*\cdot k_{34}$

$$\Rightarrow V^{(2)}_{Sk\text{-}pl} = t_2\left(1+x_2P^\sigma\right) k_{12}^*\cdot k_{34}\left(1-P^xP^\sigma P^\tau\right) =$$

$$= t_2\left(1+x_2P^\sigma\right)k_{12}^*\cdot k_{34}\left(1+P^\sigma P^\tau\right) = \vec k_{12}^*\cdot\vec k_{34}\; t_2\left(1+x_2(P^\sigma+P^\tau)+P^\sigma P^\tau\right).$$

$$= k_{12}^*\cdot k_{34}\left[t_2\left(1+\frac14(1+\vec\sigma\cdot\vec\sigma)(1+\vec\tau\cdot\vec\tau)\right)+t_2x_0\left(\frac12+\frac{\vec\sigma\cdot\vec\sigma}{2}+\frac12+\frac{\vec\tau\cdot\vec\tau}{2}\right)\right] =$$

$$= k_{12}^*\cdot k_{34}\left[\frac{t_2}{4}(5+4x_2)+\frac{t_2}{4}(1+2x_2)(\vec\sigma\cdot\vec\sigma+\vec\tau\cdot\vec\tau)+\frac{t_2}{4}\,\vec\sigma\cdot\vec\sigma\;\vec\tau\cdot\vec\tau\right]$$

we remember $\hat{\vec k}_{12} = \frac{1}{2i}(\nabla_1-\nabla_2) = \frac12(\vec k_2-\vec k_1)$ is not the "relative momentum" $\rightsquigarrow$ but the momentum in the relative frame!

$$T_1 = k_{12}^{2*}+k_{34}^2 = -\frac14\left[(\nabla_1-\nabla_2)^2+(\nabla_3-\nabla_4)^2\right] = -\frac14\left(\nabla_1^2+\nabla_2^2+\nabla_3^2+\nabla_4^2-2\vec\nabla_1\cdot\vec\nabla_2-2\vec\nabla_3\cdot\vec\nabla_4\right)$$

$$T_2 = +\vec k_{12}^{*}\cdot\vec k_{34} = \frac14\left(\vec\nabla_1\cdot\vec\nabla_3+\vec\nabla_2\cdot\vec\nabla_4-\vec\nabla_2\cdot\vec\nabla_3-\vec\nabla_1\cdot\vec\nabla_4\right)$$

for Landau parameter one must consider $\vec q = \vec k_{12}-\vec k_{34} = \vec k_3-\vec k_1 = \vec k_4-\vec k_2$ the transferred momentum goes to 0

$$q^2 = k_{12}^2+k_{34}^2-2\vec k_{12}\cdot\vec k_{34} = T_1+2T_2\,;$$
$$= (\vec k_3-\vec k_1)\cdot(\vec k_3-\vec k_2)\,\frac14$$

$\vec q_1=\vec k_1 \quad q_2=\vec k_2 \quad |q_1|=|q_2| \xrightarrow{q\to0} k_F$

$\Rightarrow \vec k_3 = \vec q_1+\vec q \quad \vec k_4 = \vec q_2+\vec q$

$\vec q_1\cdot\vec q_2 = q_1q_2\cos\vartheta = \vec k_1\cdot\vec k_2$

$\vec k_3\cdot\vec k_4 = q_1q_2\cos\vartheta + q_1q\cos\vartheta' + q_2q\cos\vartheta'' + q^2 \xrightarrow{q\to0} q_1q_2\cos\vartheta$

$$T_1 = \frac18\left(k_1^2+k_2^2+k_3^2+k_4^2-2\vec k_1\cdot\vec k_2-2\vec k_3\cdot\vec k_4\right)$$
$$\xrightarrow{q\to0}\frac18\left(2q_1^2+2q_2^2-4q_1\cdot q_2\right) = \frac{4}{8}\left(k_F^2-k_F^2\cos\vartheta\right) = \frac12 k_F^2\left(1-\cos\vartheta\right)$$

$$T_2 = \frac14\left(k_1^*\cdot k_3 + k_2^*\cdot k_4 - k_2^*\cdot k_3 - k_1^*\cdot k_4\right)$$
$$\xrightarrow{q\to0}\frac14\left(q_1^2+q_2^2-2q_1\cdot q_2\right) = +\frac12 k_F^2\left(1-\cos\vartheta\right)$$

$$V_{ph}^{(0,0)} = \sum_\ell f_\ell^{(0,0)} P_\ell(\cos\vartheta) = \frac{1}{2}k_F^2(1-\cos\vartheta)\left[\frac{3}{4}t_1 + \frac{t_2}{4}(5+4x_2)\right]$$

$$\Rightarrow f_0^{(0,0)} = \frac{k_F^2}{8}\left(3t_1 + (5+4x_2)t_2\right) \qquad f_1^{(0,0)} = -\frac{k_F^2}{8}\left(3t_1 + (5+4x_2)t_2\right) = -f_0^{(0,0)}$$

$$f_0^{(1,0)} = \frac{k_F^2}{8}\left(-t_1(1-2x_1) + t_2(1+2x_2)\right)$$

$$f_0^{(0,1)} = \frac{k_F^2}{8}\left(-t_1(1+2x_1) + t_2(1+2x_2)\right)$$

$f_1 = -f_0$ for II^{o} order

$$f_0^{(1,1)} = \frac{k_F^2}{8}\left(-t_1 + t_2\right)$$

for finite range

is symmetric

antisymmetrize the int.

$$V_{LO-ph} = t_0(1+x_0P^\sigma - y_0P^\tau - z_0P^\sigma P^\tau)\, g_a(r_1-r_2)\,(1-P^x P^\sigma P^\tau) =$$

$$= t_0(1+x_0P^\sigma - y_0P^\tau - z_0P^\sigma P^\tau)(1-P^\sigma P^\tau)\, g_a(r_1-r_2) =$$

$$= \left[t_0(1-P^\sigma P^\tau) + x_0t_0(P^\sigma - P^\tau) - y_0t_0(P^\tau - P^\sigma) - z_0t_0(P^\sigma P^\tau - 1)\right] g_a(r_1-r_2)$$

$$= \left[(t_0+z_0)(1-P^\sigma P^\tau) + (x_0+y_0)(P^\sigma - P^\tau)\right] g_a(r_1-r_2)$$

$$= \Big[\frac{3}{4}t_0(1+z_0) - \frac{1}{4}t_0(1+z_0)(1-2(x_0+y_0))\,\vec\sigma\cdot\vec\sigma - \frac{1}{4}t_0(1+z_0)(1-2(x_0+y_0))\,\vec\tau\cdot\vec\tau$$
$$- \frac{1}{4}t_0(1+z_0)(\vec\sigma\cdot\vec\sigma)(\vec\tau\cdot\vec\tau)\Big] g_a(r_1-r_2)$$

$$\Rightarrow f_\ell^{(0,0)} = \underbrace{\frac{3}{4}t_0(1+z_0)}_{\text{corresponding to } D^{(0,0)} - E^{(0,0)}} \cdot \underbrace{\alpha_\ell P_\ell(\cos\vartheta)}_{\text{expansion of } g_a \text{ in } P_\ell}$$

to find α_ℓ we transform g_a in Legendre Polynomials

$$g_a(k) = \sum_{\ell=0}^{+\infty} \alpha_\ell P_\ell(\vec k_1\cdot\vec k_2)$$

to have g in function of $\vec k$ you need to Fourier transform first